A Comparative Study of Vibration Frequency Estimates of the Surface Foundations of Wind Turbines Built on the Sand Dunes of the Ceará Coast

M. F. P. de Aguiar

A. S. Moura, S. A. D. Neto; Federal University of Ceará - Brazil
M. F. P. de Aguiar; University of Fortaleza – Brazil

EXTERNAL ARTICLE

ENGLISH

Abstract

This paper aims at presenting a comparative study of vibration frequency estimates of the surface foundations of wind turbines built on the sand dunes of the Ceará coast. Initially, some basic concepts of foundation dynamics will be addressed along with the main methods available in the literature for the calculation of vibration frequencies of rotational machines. A comparative study shall also be presented of vibration frequency estimates of surface foundations of wind turbines of two wind farms located on the coast of Ceará using different methods.

1. Introduction

Some types of machines operate at certain frequencies and often produce vibrations which may damage the structures of their construction. Fortunately, such undesirable vibrations may be controlled in such a way that they can not be fully conveyed to the construction's foundation. This is possible, in practice, by providing a certain distance from the foundation's frequency, thereby avoiding the phenomenon of resonance which may cause the damage, and can perhaps even ruin the construction. The estimate of all motion, which may be translation as well as rotation, of the machine-foundation-soil system is most commonly achieved by the method where the soil is considered homogeneous, isotropic, elastic and semi-infinite [13] and the method where the soil is replaced by linear weightless springs [3]. In order to use such methods, it is necessary to determine the dynamic parameters of the soil which is obtained from both field and laboratory tests. In practice, field tests are more desirable for this type of work because they do not present any disturbance problems during sampling.

Machines such as turbines, compressors, motors, generators and even small machines used in factories can possibly cause vibrations in the structures of constructions. In this context, the use of wind turbines deserves special attention and presents an alternative source of electric power generation in the Northeastern region of Brazil, mainly in the State of Ceará. This paper aims at comparing vibration frequency estimates of the surface foundations of wind turbines built on sand dunes of the coast of Ceará by different methods and by verifying the risk of resonance within these structures.

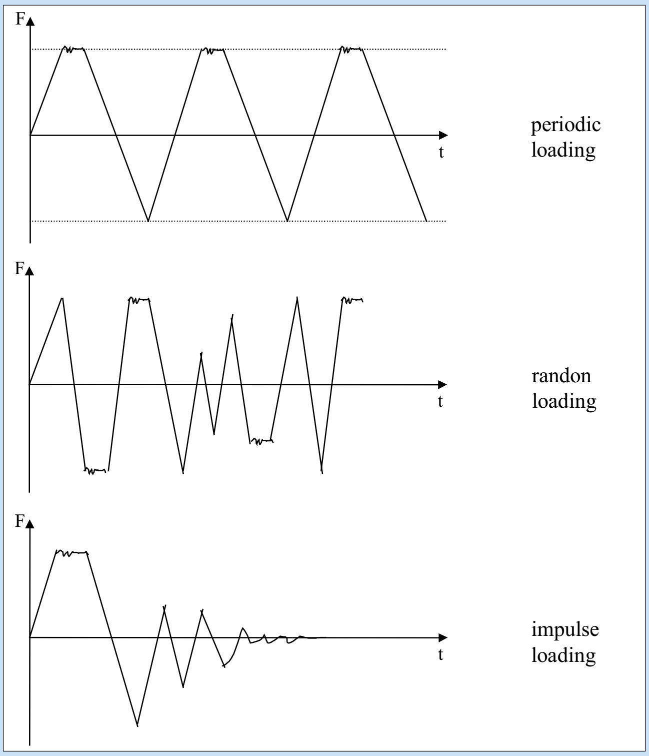

Fig. 1: Types of dynamic loading.

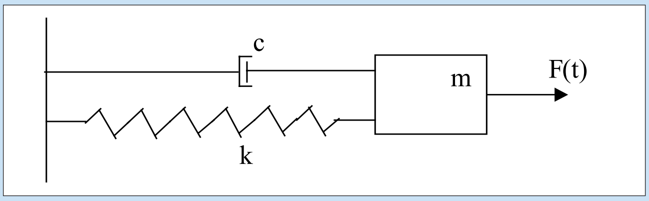

Fig. 2: Schematic representation of a system with one degree of freedom.

2. Basic Concepts in Foundation Mechanics

The static behavior of a structure is, as a matter of fact, a particular case with no acceleration in terms of dynamic behavior. Loading may be characterized as dynamic when it varies in time, in value and/or direction.

Conventional buildings, during and after their construction, present a variation in mass. However, this takes place in small increments which usually do not disturb the static balance between the weight of the building and the soil´s reaction. The loading speed in a dynamic phenomenon depends greatly on the size of the soil mass involved [7]. From that point onward, common samples used in laboratories only have reactions characterized as dynamic starting from 25 Hz. On the other hand, large earth dams may undergo forces of inertia with important frequencies of around 0.5 Hz.

Dynamic events in Geotechnics are, generally, related to machine foundations, wave propagation on the soil and loading of piles. Machines with turbines, compressors, motors, generators and even small machines used in factories may trigger important vibrations in the construction´s structures [11]. Thus, in this context, the importance of the presence of vibrations in the structures of wind turbines receives special attention.

The operation of rotational machines may bring about an unbalance of forces which accelerate the machine-structure-foundation set. If the motion generated is periodic, the set oscillates, in the case of excessive vibrations, they may hinder the functionality or, may also hinder the use of a given construction.

Dynamic loading can be classified as periodic, random and impulse. Fig. 1 displays the configuration of the types of dynamic loadings. In periodic loading, there is repetition in regular time intervals. In the case of random loading, the load variation presents no regularity. Impulse is the case where loading is applied one single time. The operation of rotational machines is a typical case of periodic loading and the waves in off-shore structures of random loading. Impulse takes place in impact loads much like in protectors used in the anchoring of ships.

The machine-structure-foundation system is comprised of a complex vibrating system with a high degree of freedom and natural frequencies which is subject to the phenomenon of resonance. Degree of freedom means the number of displacement directions in which a given system is able to oscillate in. Structural displacement results in inertia forces. However, the displacement itself influences the size of the force, therefore, it is said that there is a closed cycle of cause and effect. The formulation of the problem may be established through differential equations. Deterministic analysis allows for a historical evaluation of the displacement in time which took place in the structure due to dynamic loading. The mathematical expressions which define dynamic displacement are called equations of motion. Clough and Penzien presented three methods which may be utilized in the formulation of equations of motion: Direct equilibrium according to d'Alembert's principle and Hamilton´s Principle of Virtual Work [4].

The simplest representation of a structure is comprised from mathematical models of degrees of freedom (Fig. 2). In a degree of freedom system, motion is in only one direction. Thus, only one coordinate is necessary to totally define the position of a given mass considered to be centered. This mass is submitted to external loading, F(t), which varies in time and produces the response of the system.

The components of a system with one degree of freedom are: a mass, the elastic properties, the damping mechanisms and the external force of loading. The spring, of stiffness k, without weight, supplies the elastic resistance to the displacement and to the damper, c, represents energy dissipation. Applying the equilibrium of forces which act upon the mass in Fig. 2 in the direction of the displacement, the following equation is produced:

$$F_i + F_{am} + F_e = F(t) \quad (1)$$

Where: F(t) is the external force, $F_i = m.\frac{d^2x}{dt^2}$ is the inertial force, $F_{am} = c.\frac{dx}{dt}$ is the dampening force and F_e = k.x is the elastic force. The replacement of these expressions in equation 1 produces the equation of motion of a forced system with one degree of freedom:

$$m.\frac{d^2x}{dt^2} + c.\frac{dx}{dt} + k.x = F(t) \quad (2)$$

Where, m is the foundation mass, c is the dampening constant, k is the elastic constant and F(t) is the external force. For cases of bodies undergoing natural oscillations, in other words, oscillations occur when a body is displaced and after abandoned, equation (2) becomes:

$$m.\frac{d^2x}{dt^2} + c.\frac{dx}{dt} + k.x = 0 \quad (3)$$

Motion of this type is called damped harmonic motion. This motion provides a solution expressed in terms of sine and cosine functions and amplitude of oscillation gradually decreased by friction. If there is no damping (c = 0), the motion is called free or simple harmonic motion. For models with various degrees of freedom, the motion formula is analogous to that of the motion of a system of one degree of freedom. In this case, a break-down of the mass is performed whose forces are associated to each degree of freedom present.

3. Calculation Methods of Machine Foundations

The calculation methods of machine surface foundations may be grouped as empirical methods, soil as an elastic semi-space, soil as a set of linear weightless springs or numerical methods. Empirical methods are more adequate in a preliminary analysis [5]. Methods which consider the soil as an elastic semi-space and as a set of linear weightless springs should only be utilized in cases of very low levels of deformation and numerical methods are more suited for cases where there are numerous degrees of freedom.

3.1. Empirical Methods

Included in the group of empirical methods is the method from the German Research Society for Soil Mechanics [15], methods which take into account the soil mass, Tschebotarioff and Ward´s method [16] as well as Alpan´s method [2]. The DEGEBO method [15] stems from research performed in the beginning of the century, in Berlin, through a mechanical oscillator with four eccentric masses acting in the vertical and torsional modes. As a result from this research, a series of frequency characteristics was published for a broad range of soils. In the methods that take soil mass into account, the soil acts as a spring which vibrates together with the foundation. The main difficulty found in the use of these methods resides in the quantification of the soil´s weight in vibration. Tschebotarioff and Ward´s method [16] proposes to obtain the natural frequency, f_n, as a function of the foundation´s base area, A, the foundation´s weight plus the machine, P_v, and the reduced natural frequency, f_{nr}, in such a way that:

$$f_n = \sqrt{\frac{A}{P_v}}.f_{nr} \quad (4)$$

Alpan´s method [2] proposes the following relation:

$$f_n = \frac{a'}{\sqrt{P_v}}.A^{0,25} \quad (5)$$

Where: f_n is the natural frequency, P_v is the foundation´s weight plus the machine, A is the foundation´s area and a' is a parameter which is a function of the soil type. Empirical methods, such as Alpan´s method [2], should only be used in preliminary designs in order to verify the incidence of resonance [14].

3.2. Methods which Consider the Soil as Elastic Semi-space

The theory of elastic semi-space studies the vibrations of a vibrating rigid foundation at the surface of a semi-space comprised of dimensions which are infinite, homogeneous and isotropic, whose stress-strain relationships are defined by two constants: the shear modulus and Poisson´s ratio. The method is analytical and accepts the hypothesis of small deformations which is necessary to consider the linear elasticity of soils, and considers the loss of energy of the soil mass due to the dampening effect, based on the wave propagation theory through an elastic media, thereby accepting various simplifying hypotheses to facilitate the mathematical solution of the problem. The main difficulty found in the use of the elastic semi-space methods is in determining the soil parameters, in other words, the shear modulus (G) and Poisson´s coefficient (υ) [9].

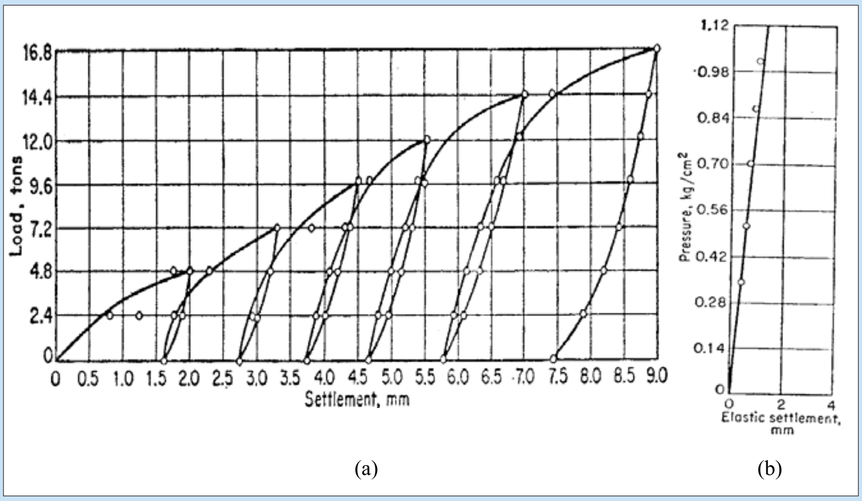

Fig. 3: a) Results from a cyclic plate load test b) Graphic plot of c_z [3].

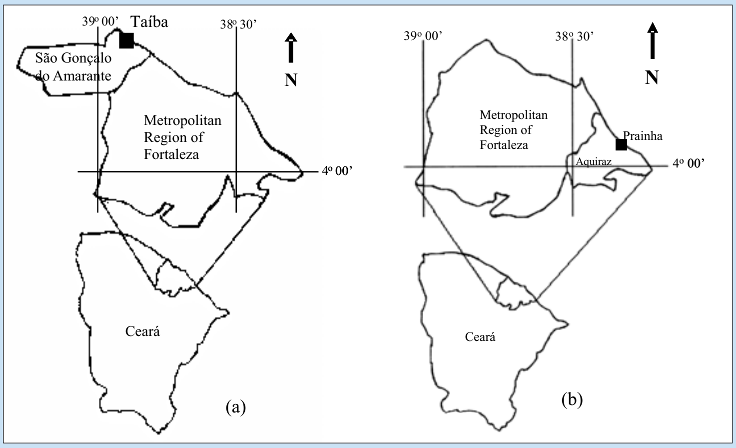

Fig. 4: Location of the wind turbines a) Municipality of São Gonçalo do Amarante b) Municipality of Aquiraz.

The following is a presentation of some methods which consider the soil as an elastic semi-space. In Hsieh´s method [6], the vertical motion of a rigid foundation is given by:

$$m.\frac{d^2z}{dt^2}+\sqrt{G.\rho}.r_o.F_2.\frac{dz}{dt}-G.r_o.F_1.z=F \quad (6)$$

In this way, the damping coefficient, c, and the stiffness coefficient (elastic constant), k, are obtained by:

$$c=\sqrt{G.\rho}.r_o.F_2 \quad (7)$$

$$k=G.r_o.F_1 \quad (8)$$

Where: F_1 e F_2 are displacement functions for the vertical vibration in the interval from $0<a_o<1.5$.

Lysmer and Richart´s proposal, lists the methods which consider soil as an elastic semi-space with a damped oscillating system [8]. The proposed constants in the method are:

$$k_z=\frac{4.G.r_o}{1-\upsilon} \quad (9)$$

$$c=\frac{3,4.r_o^2}{1-\upsilon}.\sqrt{G.\rho} \quad (10)$$

Nagendra and Sridharan proposed, from modified displacement functions, regardless of Poisson´s coefficient (υ), the following expressions for the case of uniform pressure distributions [12]:

$$k_z=\frac{\pi.G.r_o}{1-\upsilon} \quad (11)$$

$$c=\frac{2,117.r_o^2.\sqrt{G.\rho}}{1-\upsilon} \quad (12)$$

3.3. A Method which Considers the Soil as a Set of Linear Weightless Springs [3]

Following the method where the soil is replaced by a system of linear weightless springs, the damping effect as well as the effect of the participation of the spring´s mass, are neglected. Although damping has a considerable effect on reso-

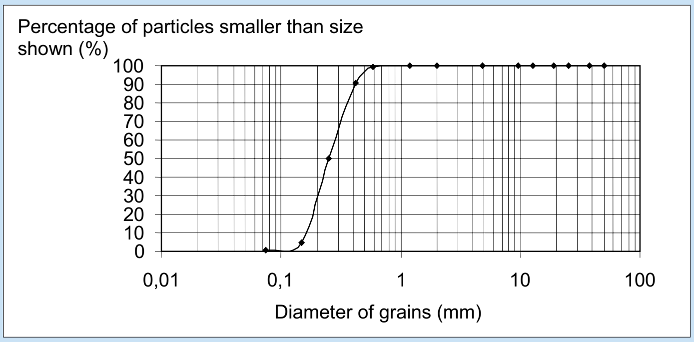

Fig. 5: Typical grain-size-accumulation curve of the material.

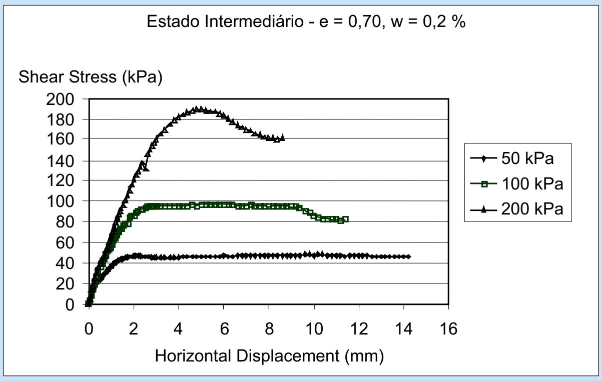

Fig. 6: Shear Stress x Horizontal Displacement Curve (e = 0.70, dry sample).

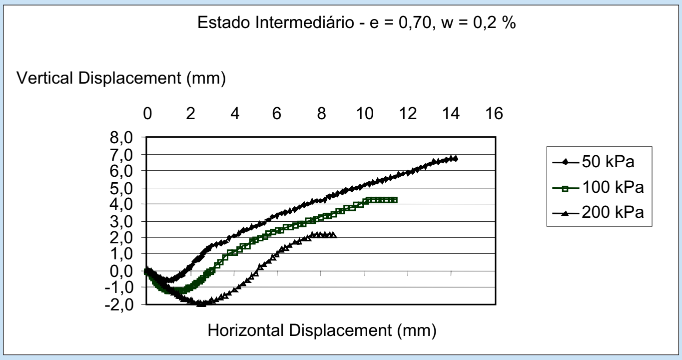

Fig. 7: Vertical Displacement x Horizontal Displacement Curve (e = 0.70, dry sample).

nance amplitudes, thereby decreasing the amplitudes, it may be neglected, in favor of security, as long as the resonance range is avoided in the foundation´s designs [15].

Despite the fact that Barkan´s method [3] neglects the damping effects as well as soil mass participation, its use is quite simple and able to perform forecasts quite accurate in terms of the foundation´s real behavior [14].
For vertical vibrations, applying Hooke´s law, note the following:

$$F_z = -k_z.z \qquad (13)$$

If there is no damping and making F(t) = P_z.sen ω".t, leads to:

$$m\frac{d^2z}{dt^2} + k_z.z = P_z.sen\omega''.t \qquad (14)$$

Which is, the equation of the vertical motion of a forced vibration, without any damping.

Making $c_z = p/z_e$, where c_z is the uniform elastic compression coefficient, p is the uniform compression pressure and z_e is the elastic settlement and not forgetting that the pressure, p, is equal to the relationship between the vertical force, F_z, and the foundation´s base area, A, which leads to:

$$c_z = \frac{F}{A.z_e} \qquad (15)$$

But applying Hooke´s law, leads to:

$$c_z = \frac{k_z}{A} \qquad (16)$$

Where: k_z represents soil stiffness, c_z is an elastic compression coefficient and A is the area of the foundation´s base. Replacing in the vertical motion equation of the forced vibration without damping, the following holds true:

$$m\frac{d^2z}{dt^2} + c_z.A.z = P_z.sen\omega''.t \qquad (17)$$

With the use of a resonance block, the determination of c_z is achieved by a vibration analysis of a block supporting an oscillator in order to determine the resonance frequency.

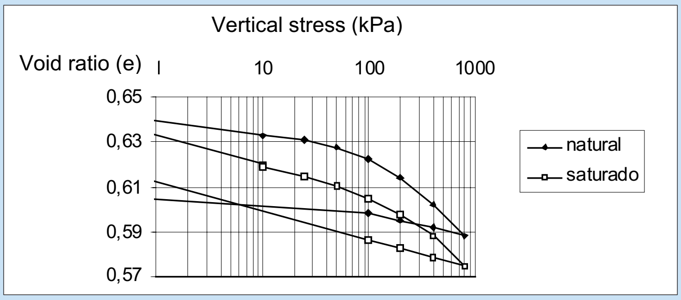

Fig. 8: Results of the double oedometric test.

Fig. 9: Performance of penetration surveys.

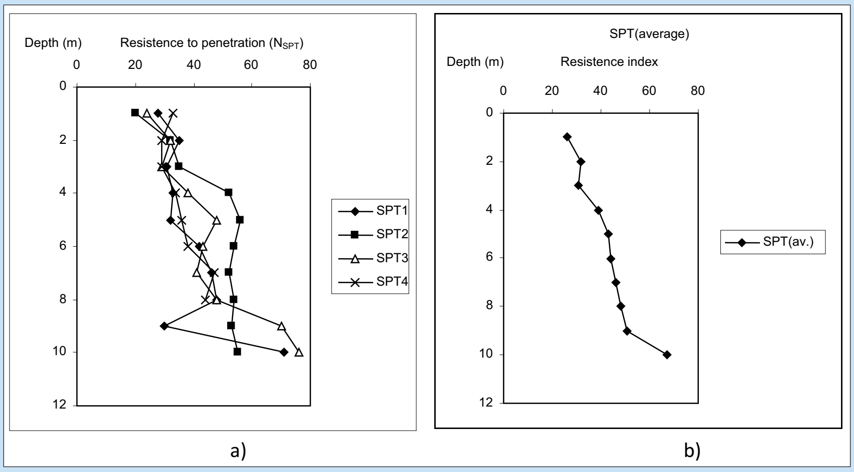

Fig. 10: a) Comparison between the resistance index values for surveys SPT1, SPT2, SPT3 and SPT4. b) Average resistance profile.

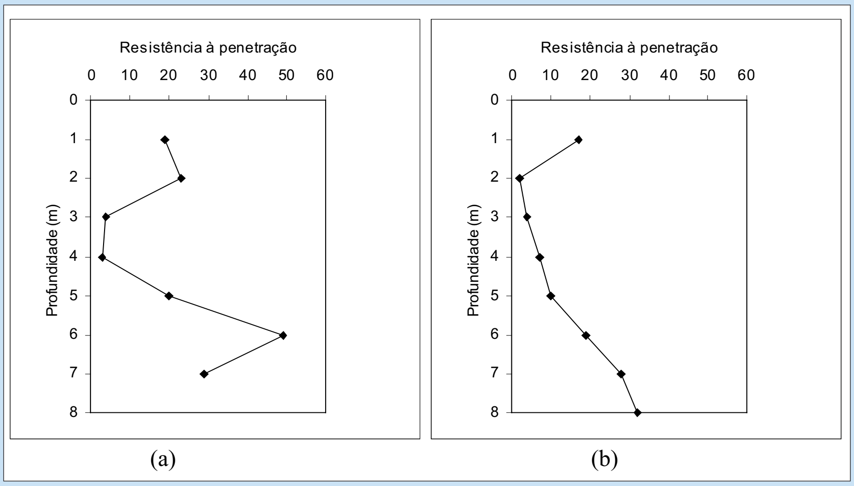

Fig. 11: Penetration resistance profiles a) SPT1 b) SPT2

The mechanical oscillator provides vertical excitation through unbalanced masses with variable operation frequency, from where the frequency may be obtained and which corresponds to the maximum amplitude. Assuming that the natural frequency is the resonance frequency, the expression of the natural frequency provides the elastic compression coefficient (c_z), thus:

$$c_z = \frac{4\pi^2 f_{nz}^2 .m}{A} \tag{18}$$

Where: f_{nz} is the natural frequency corresponding to the maximum amplitude, m is the foundation and machine´s mass, and A is the area of the foundation´s base. Finally, through cyclic plate load tests, it is possible to determine c_z by supporting a plate over the soil and submitting it to repeated cycles of loading and unloading which are increased at every stage. For each unloading stage, there shall be a part of the elastic settlement and another which is residual. From the relationship between the soil pressure and the elastic settlement it is possible to obtain the value of c_z. (Fig. 3).

Generally, the participation of the soil mass in the foundation´s vibrations does not usually exceed 23% of the total mass (foundation and machine). Where the natural frequency is a function of the square root of the mass, the calculation of these frequencies, including the soil mass, would not alter more than 10% [3].

4. Location of the Wind Turbines

The areas which were studied correspond to the beach areas of Taíba and Prainha. Taíba beach is located in the municipality of São Gonçalo do Amarante, which is limited to the East by the Metropolitan Region of Fortaleza, MRF, and is around 60 km from the State Capital. Access may be via state highway CE 085 or federal highway BR 222 (Fig. 4). Prainha beach is located in the municipality of Aquiraz, approximately 20 km from the Capital, and is actually part of the Eastern portion of the MRF (Fig. 5).

5. Data Collection

The necessary data for this study, regarding the wind turbines of Praia da Taíba (Municipality of São Gonçalo do Amarante) were obtained from Moura [10] and the data concerning the turbines of the Prainha wind farm (Municipality of Aquiraz) were collected from local companies. For Taíba, the results were obtained from geotechnical laboratory tests (characterization and special testing) as well as field tests (SPT and CPT), as well as the geometric characterization of the existing wind turbines in the location. As far as Prainha is concerned, data was only collected from Standard Penetration Tests (SPT) as well as the geometric characterization of the existing wind turbines in the location.

5.1 Geotechnical Tests Performed for Praia da Taíba

The behavior of the soil at Taíba Beach was determined from a laboratory test program (characterization and special) of a battery of field tests, penetration and pressuremeter tests.

The laboratory investigation comprised of a battery of characterization tests and other special tests. The characterization component included granulometric and absolute specific gravity of the solid substance, consistency limits as well as maximum and minimum void ratio. Additionally, "*in situ*" natural moisture and natural density profiles were traced. Fig. 5 shows a typical grain-size-accumulation curve of the material surveyed.

According to the Highway Research Board´s classification (HRB), all the samples analyzed in the sub-group of soils A-3, corresponding to fine sand. Following the Unified Soil Classification System (USCS) all samples belong to the group SP, in other words, poorly graded sand, with little quantity of fines.

Aiming at obtaining characteristic values of the resistance parameters of the soil surveyed, two direct shear test programs were performed on statically compacted sand samples, one under natural conditions, 3% moisture, and another under completely dry conditions. In Fig. 6 and 7 are the results obtained with direct shear testing performed in an intermediate state (e=0.70) on dry samples.

The values obtained from ϕ as a function of the level of empty spaces for the test samples in a wet state made it possible to plot a graph and was extrapolated to field conditions. Determining the friction angle of the soil through out the depth, contrary to what was previously thought. In fact, the friction angle estimate did not change according to the depth and was kept constant at 43º.

In order to estimate the oedometric modulus value of the soil under study, as well the saturation effect of the same soil, a double oedometric test was performed. This method is widely used for the assessment of the collapse of non-saturated soil and is performed by two tests. In order to do so, two samples are taken under the same initial conditions. One of the samples is previously saturated and afterwards is submitted to standard loading stages. The other sample is tested under natural conditions, with constant moisture. Fig. 8 displays curves versus σ_v of the oedometric double tests performed.

Soil compressibility was determined as being low, in other words, the soil in this study presented a high level of stiffness. In the oedometric test performed with natural moisture, the compression index (C_c) was obtained and represented by the value of 0.043 and with a recompression index (C_r) of 0.011. The soil tested under saturated conditions presented a slight decrease in compressibility, in this case C_c and C_r presented values of 0.038 and 0.013, respectively.

Penetration testing is the most recognized geotechnical research method used in Brazil. Resistance to penetration is obtained from tests which indicate the soil resistance which allows for the extraction of samples for soil characterization purposes. The resistance to penetration values obtained in the test is usually used in foundation designs.

A total of 4 penetration surveys were performed, according to the Brazilian Standard NBR 6484/01 [1], until a depth of 10m using a manual auger. Fig. 9 illustrates the performance

Fig. 12: General view of the Taíba wind turbine.

of penetration surveys and Fig. 10 shows the comparison between resistance to penetration indexes from the four surveys performed as well as the average resistance profile.

5.2 Geotechnical tests performed for Praia da Prainha

The soil at Prainha beach corresponds to fine white sand, varying from soft to very compact. Two penetration surveys (SPT) were collected at a distance of 400m from each other. The resistance profiles of the two samples are displayed in Fig. 11a and 11b. On average, it was estimated that for the resistance index, $N_{SPT(av)}$=17. It is worth mentioning that the water table level NA was not found.

6. Characterization of Wind Turbines

The wind turbines of Taíba and Prainha are equal, all manufactured by Wobben Windpower/Enercon, model E-40, rated capacity of 500 kW, rotor diameter of 4.2m, hub height of 46.2m, with active blade pitch control, clockwise rotation, with three blades for each turbine, each blade with a length of 18.9m and weighing 13 kN. The blades are manufactured with reinforced fiberglass and epoxy. The generator has a horizontal shaft and weighs 136 kN.

The foundations for the wind turbines are comprised of square footing made of reinforced concrete, measuring 9m on the side and 1.5m in height. The towers measure 44m in height and are made of 2.54cm thick steel and weigh 359 kN. The tower diameter at the base measures 2.5m and 1.2m at the maximum height. The nacelle is made of fiberglass, and has a diameter of 4.4m, length of 6.7m and weighs 129 kN.

Considering that the specific weight of the reinforced concrete is 25 kN/m^3, the weight of the foundation for each wind turbine is approximately 3.038 kN. Adding the weight of the tower, 359 kN, the nacelle, 129 kN, the generator 136 kN and the three blades 39 kN, an estimate of the total weight is around 3.700 kN, for each wind turbine. Fig. 12 and 13 show a general view of a wind turbine located at Taíba beach along with some details of the blade.

7. Vibration Frequency Estimates and Comparison of Results

The vibration frequency estimates of the wind turbines of Taíba and Prainha were performed following some methods disseminated in the literature, three empirical methods (DEGEGO [15], Tschebotarioff and Ward [16] and Alpan [2]), two other methods which take into account the soil as an elastic semi-space (Lysmer and Richart [8] and Nagendra and Sridharan [12]). Besides these methods, a classic method was also utilized which considers the soil as a set of linear weightless springs [3]. Tab. 1 shows the natural vibration frequency estimates of the wind turbines of Taíba and Prainha. Fig. 14 displays a comparison of the estimates performed.

According to Tab. 1, it is possible to observe a broad range of natural variation frequency estimates, ranging from 331 to 1529 rpm. With the exception of the method from the German Research Society for Soil Mechanics where the reliability of the estimates is questionable, among other factors such as the influence that the contact area has on the vibration frequency [15], the empirical methods provide the lowest estimates. A special mention is given to the natural

Fig. 13: Details of the Taíba wind turbine blade.

Method	f_n (rpm)	
	Taíba	Prainha
DEGEGO [15]	1455	1455
Tschebotarioff and Ward [16]	385	385
Alpan [2]	404	404
Lysmer and Richart [8]	1394 to 1507	890 to 912
Nagendra and Sridharan [12]	1235 to 1335	789 to 808
Barkan [3]	1529	489

Tab. 1: Natural vibration frequency estimates (f_n) for the foundations of the wind turbines of Taíba and Prainha.

vibration frequency estimates derived by the empirical methods of Tschebotarioff and Ward [16] and Alpan [2] which were quite robust and ranged from 331 to 404 rpm. On the other hand, the elastic semi-space methods estimated the greatest natural vibration frequency values, for Taíba Beach, on average ranged from 1285 to 1451 rpm and, for Prainha Beach, ranged from 799 to 912 rpm. The Barkan proposal that considers the soil as a set of linear weightless springs [3], the natural frequency was estimated to range from 1529 to 489 rpm for the foundations of both Taíba and Prainha, respectively.

The vibration frequencies estimated by the two empirical methods for both situations analyzed were equal. Since the support soil of the foundations present distinct geotechnical characteristics, proven by the large difference in the resistance index (Fig. 10 and 11), it becomes important to observe the low reliability of these methods.

The natural vibration frequency estimated by the methods which consider the soil to be an elastic semi-space (Lysmer and Richart [8] and Nagendra and Sridharan [12]) presented differences in the order of 37%. The greater frequency values were recorded for the Taíba wind turbines. This outcome makes sense, considering that stiffer soil usually presents lower amplitudes and greater vibration frequencies.

Comparing the f_n values for the Taíba and Prainha foundations, it becomes evident that the Barkan method [3] presented major differences. This fact took place because the estimates for Prainha used the average resistance index (N_{SPT}) from penetration tests (SPT) and, for Taíba, cyclic pressuremetric modulus values (E_{ur}) were used and were obtained from pressuremetric tests (PMT). Thus, the shear modulus (G) utilized for the vibration frequency estimates obtained by different ways, correspond to distinct deformation levels as well. Tab. 2 shows a comparison of the estimates performed for the natural vibration frequencies (f_n) and damped (f_n') for the foundations of Taíba Beach. The damped frequency estimates (f_n') are directly linked to the damping ratio (D), in other words, the greater the D value, the greater is the difference between f_n and f_n'.

8. Conclusion

The work behind this paper allowed the authors to reach the following conclusions:

- The range in variations of the natural vibration frequency estimates was quite wide, varying from 331 to 1529 rpm;
- According to the natural vibration frequency estimates of the Taíba and Prainha wind turbines, it becomes evident that the empirical methods estimate the lowest values and that the elastic semi-space methods estimate the highest values;
- The vibration frequency estimates from the empirical methods proved to be somewhat unreliable, where both situations analyzed presented distinct geotechnical soil characteristics, yet the estimated frequencies were equal;
- The elastic semi-space method from Barkan [3] provides distinct f_n value estimates when compared to the empirical methods, since the first method considers soil deformability characteristics of the foundations;

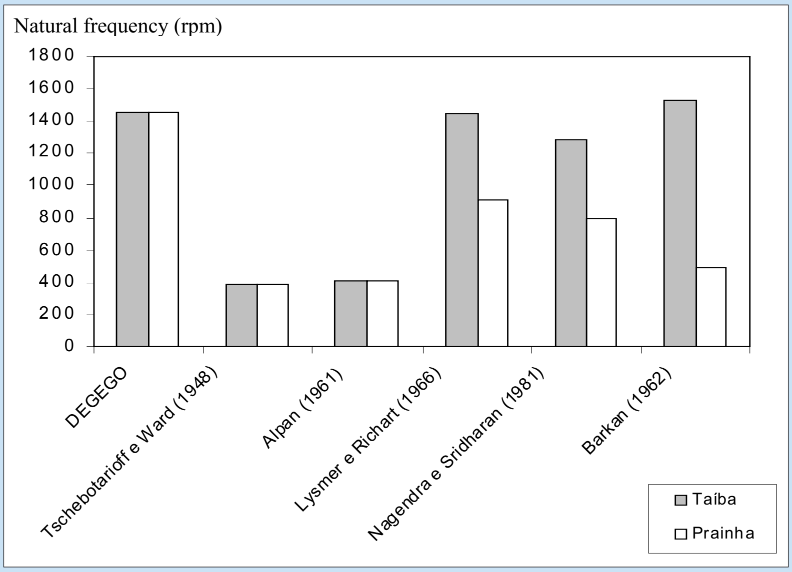

Fig. 14: Comparison of natural vibration frequency estimates (f_n) for the foundations of the wind turbines of Taíba and Prainha.

Method	fn (rpm)	f'n (rpm)
DEGEGO [15]	1455	-
Tschebotarioff and Ward [16]	385	-
Alpan [2]	404	-
Lysmer and Richart [8]	1394-1507	1302-1407
Nagendra and Sridharan [12]	1235-1335	1018-1100
Barkan [3] - no damping	1529	-
Barkan [3] - with damping	-	1162

Tab. 2: Comparison of the estimates performed for the natural vibration frequencies (f_n) and damped (f_n') for the foundations of Taíba Beach.

- Elevated differences between f_n estimates by the elastic semi-space method of Barkan [3] are mainly due to the different tests utilized, and consequently, the different levels of deformation of each one;
- The damped frequency estimates (f_n') are directly linked to the damping ratio (D).

References

[1] ASSOCIAÇÃO BRASILEIRA DE NORMAS TÉCNICAS (2001). NBR 6484/01 - Execução de Sondagens de Simples Reconhecimento dos Solos. Rio de Janeiro.

[2] ALPAN, I. (1961) Machine Foundations and Soil Ressonance. Geotechnique, vol. 11, pp. 95-113.

[3] BARKAN, D.D. (1962). Dynamics of Bases and Foundations. McGraw-Hill Book Company, New York, 434 p.

[4] CLOUGH, P. e PENZIEN J. (1975). Dynamics of Structures. McGraw-Hill Book Co. Singapore. 634 p.

[5] COSTA, A. A. (1988). Análise de Fundações de Máquinas Sujeitas a Excitações Verticais. Tese de Mestrado em Geotecnia da COPPE/UFRJ. Rio de Janeiro. 184 p.

[6] HSIEH, T. K. (1962). Foundation Vibrations. Proceedings of Institution of Civil Engineers, Vol. 22, pp. 211-226.

[7] LAMBE, T. W. e WHITMAN, R. V. (1976). Mecanica de Suelos. Editorial Limusa. México. 582 p.

[8] LYSMER, J. e RICHART, F. E. (1966). Dynamic Response of Footings to Vertical Loading. Journal of the Soil Mechanics and Foundations Division, Proc. ASCE, Vol. 92, No. SM1, January, pp. 65-91.

[9] MORGAN, J. R. e MOORE, P. J. (1968). Application of Soil Dynamics to Foundation Design. In: Soil Mechanics, Selected Topics. Edited by J.K. Lee. Butterworth & Co. (Publishers) Ltd. pp. 465-527.

[10] MOURA, A. S. (2007). Avaliação de Metodologias de Projeto de Fundações Superficiais de Aerogeradores Assentes em Areia de Duna. Tese de Doutorado, Departamento de Engenharia Civil, Universidade de Brasília, 323 p.

[11] MOURA, A. S. e SILVA FILHO, F. C. (2004). Ensaios Especiais para Otimização de Aerogeradores. Revista Coelce Conhecimento. Fortaleza. Ano I, nº 01. pp. 37-41.

[12] NAGENDRA, M. V. e SRIDHARAN, A. (1981). Response of Circular Footings to Vertical Vibrations. Journal of the Geotechnical Engineering Division, Proc. ASCE, Technical Notes, Vol. 107, No. GT7, July, pp. 989-995.

[13] RICHART, F. E., Jr., HALL, J. R., and WOODS, R. D. (1970). Vibrations of Soils and Foundations. Prentice-Hall, Inc., Englewood Cliffs, New Jersey, 414 p.

[14] SRINIVASULU, P. and VAIDYANATHAN, C. V. (1978). Handbook of Machine Foundations.TATA McGRAW-HILL Publishing Company Ltda. New Delhi. 238 p.

[15] TSCHEBOTARIOFF, G. P. (1978). Fundações, Estruturas de Arrimo e Obras de Terra. McGraw-Hill do Brasil. São Paulo. 513 p.

[16] TSCHEBOTARIOFF, G. P. e WARD, E. R. (1948). The Resonance of Machine Foundations and the Soil Coefficients with Affect it, Proc. 2nd Intern. Conf. on Soil Mech & Found. Engng, Rotterdam, Vol I, pp. 309-13.